## Object in Equilibrium

## Equipment

- Computer with spreadsheet software
- Circular force table, 3 pulleys and strings
- 3 mass hangers and a mass set


## Objectives

- To verify the condition of static equilibrium.


## Introduction

Scalars are quantities completely specified by a magnitude. Examples include temperature, mass, and time. Vectors are quantities represented by both a magnitude and a direction. Examples include displacement, velocity, and force.

While working with scalar quantities one can just algebraically add, subtract, multiply or divide numbers. However, when working with vectors one need to account for the direction of the vector and therefore a simple addition or subtraction would be incorrect. To add vectors, one should follow the rules of vector addition. A vector quantity could be expressed in two general ways: magnitude-direction form (or polar coordinates) and vector components form. Between the two, the vector components form is probably more convenient for the manipulations. A vector could be resolved into components through trigonometry where each component is a projection of the vector in each dimension and is expressed as a product of the vector's magnitude and a corresponding trigonometrical function, $\boldsymbol{\operatorname { s i n }}$ or $\boldsymbol{c o s}$. If the component is adjacent to the angle in use, it is a cos function; however, if the component is opposite to the angle in used, then it is a $\boldsymbol{\operatorname { s i n }}$ function. The components of different vectors but projected in the same dimension are in the same direction and could be manipulated just as scalars. When the identical manipulations are done to each set of components, it produces the corresponding components of the resultant. In turn, the magnitude and direction of the resultant could be calculated from the components by the means of Pythagorean Theorem.

A force (typically represented by the symbol $\vec{F}$ ) is an example of a vector quantity. An object is in equilibrium when the sum of all forces acting on the object is zero. In equilibrium, a stationary object will remain stationary. To verify equilibrium, it is necessary to add all the forces acting on the object following the rules of vector addition. If the vector sum of all forces is about zero and the stationary object remains at rest, the condition of equilibrium is satisfied.

Before starting this experiment, improve your knowledge of the rules of vector manipulations. Your review should include the addition of vectors and two ways of representing vectors: vector components and magnitude-direction. Spreadsheet may help with the task, learn Vector Addition in a Spreadsheet from https://www.youtube.com/watch?v=KqjZd6yyslA . Remember that when calculating components in Excel, the angle should be converted to radians.
Example: $=\mathrm{B} 2 * \cos$ (radians (C2)) .

## Experimental Setup

The Force Table, see Figure 1, is a round platform with a protractor printed on it. In the center is a small plastic disk attached to a few strings that are placed over pulleys clamped around the edge. A mass hanger of 5 grams could be suspended from the end of each string below the pulleys and additional small masses could be placed on the hanger. Various parts of the experiment require different number of hangers in use. The disk serves as an object of study and is in equilibrium if it is centered.


Figure 1: Force Table setup, with 3 equally spaced pulleys that have equal masses suspended from them. (Image credit: PASCO)

## Part 1: Object submitted to two forces

1. If only one force is applied to the object, the object will not be in an equilibrium state. However, if two forces are acting on the object, they might cancel each other out resulting in a net force of zero magnitude.
2. Hang two mass holders from strings that pass over pulleys. (Do not hang anything on the third string, just place it on the table.) Adjust the position of each pulley until the disk is in equilibrium and is perfectly centered.
3. Record the magnitude and direction of each force in Table 1 (see below). Use value of the mass of the holder as the magnitude. Do not include units in the recordings.
4. Calculate x - and y -components of each force; afterward, total each corresponding column to find $x$ - and $y$ - components of the Resultant.
5. Calculate the magnitude of the Resultant using the Pythagorean Theorem. For $x$ and $y$, use the components of the Resultant, not the magnitudes of the two force vectors !!!
6. Calculate the direction of the Resultant using the function = DEGREES (ATAN ( $\mathrm{y} / \mathrm{x}$ ) ) or $=\operatorname{DEGREES}(\operatorname{ATAN} 2(x, y))$. For $x$ and $y$, use the components of the Resultant, not the magnitudes of the two force vectors!!!

| Force | Magnitude | Direction ( ${ }^{\circ}$ ) | x-component | y-component |
| :---: | :---: | :---: | :---: | :---: |
| Force $\vec{F}_{1}$ |  |  |  |  |
| Force $\vec{F}_{2}$ |  |  |  |  |
| Resultant |  |  |  |  |

Table 1. Measurements and calculations for two force.

## Part 2: Object submitted to three forces; two are of equal magnitude

 and perpendicular.1. Suspend another mass holder from the third string.
2. Set up Force 1 and Force 2 perpendicular to each other and Force 3 at a random position. At this point, Do Not balance the disk.
3. Record the direction and magnitude of only Force 1 and Force 2 in Table 2. Use value of the mass of the holder as the magnitude. Do not include units in the recordings.
4. Calculate $x$ - and y-components of Force 1 and Force 2; afterward, total each corresponding column to find x - and y - components of the Resultant.
5. Calculate the magnitude and direction of the Resultant. See Part 1, steps 5-6.
6. Predict what Force 3 or Neutralized is necessary to balance the Resultant and achieve equilibrium. (Hint: the components of the Neutralizer should be opposite to the components of the Resultant.) Calculate the magnitude and direction of the resultant just as you did with the Resultant.
7. Set Force 3 as predicted (add mass if necessary).
8. If necessary, adjust Force 3 till the disk is in equilibrium. Record the actual values of the magnitude and direction that best balance the disk.
9. Calculate the \% Difference between predicted and actual values of the magnitude and direction of the Neutralizer. Since both values are determined through the experiment,

$$
\% \text { Difference }=\frac{\text { ABS }(\text { Predicted }- \text { Actual })}{\text { average }(\text { predicted, actual })}(100 \%)
$$

| Force | Magnitude | Direction ( ${ }^{\circ}$ ) | x-component | y-component |
| :---: | :---: | :---: | :---: | :---: |
| Force $\vec{F}_{1}$ |  |  |  |  |
| Force $\vec{F}_{2}$ |  |  |  |  |
| Resultant |  |  |  |  |
| Predicted <br> Neutralizer $\overrightarrow{F_{3}}$ |  |  |  |  |
| Actual <br> Neutralizer $\overrightarrow{F_{3}}$ |  |  |  |  |
| \% Difference |  |  |  |  |

Table 2. Measurements and calculations for three forces

## Part 3: Object submitted to three Non-perpendicular forces; two are of equal magnitude.

1. Place Force 1 at $65^{\circ}$ and Force 2 at $115^{\circ}$. Place Force 3 at a random position. At this point, Do Not balance the disk.
2. Record the direction and magnitude of only Force 1 and Force 2 in Table 3. Use value of the mass of the holder as the magnitude. Do not include units in the recordings.
3. Calculate $x$ - and y-components of Force 1 and Force 2; afterward, total each corresponding column to find x - and y - components of the Resultant.
4. Calculate the magnitude and direction of the Resultant.
5. Predict what Force 3 or Neutralized is necessary to balance the Resultant and achieve equilibrium. Hint: components of Neutralizer should be opposite to the components of the Resultant. See Part 1.
6. Set up Force 3 as predicted and adjust Force 3 till the disk is in equilibrium. Record the actual magnitude and direction that best balanced the disk.
7. Calculate $\%$ difference between predicted and actual values of the magnitude and direction of the Neutralizer. See part 2, step 9.
8. Investigate if the balance will hold when you move Forces 1 and 2 further apart (to $50^{\circ}$ and $130^{\circ}$ ). If not, investigate what should be done about Force 3 to restore the balance. Include the result of your investigations in the abstract.

| Force | Magnitude | Direction ( ${ }^{\circ}$ ) | x-component | y-component |
| :---: | :---: | :---: | :---: | :---: |
| Force $\vec{F}_{1}$ |  |  |  |  |
| Force $\vec{F}_{2}$ |  |  |  |  |
| Resultant |  |  |  |  |
| Predicted <br> Neutralizer $\overrightarrow{F_{3}}$ |  |  |  |  |
| Actual <br> Neutralizer $\overrightarrow{F_{3}}$ |  |  |  |  |
| \% Difference |  |  |  |  |

Table 3. Measurements and calculations for three none perpendicular forces

## Part 4: Object submitted to three non-perpendicular forces, each of different magnitude.

1. Place Force 1, Force 2, and Force 3 at random positions. Add 10 g to Force 1, 20 g to Force 2, and 25 g to Force 3 .
2. Adjust positions of all pulleys to balance the disk while making sure that no two forces are perpendicular to each other.
3. Record the direction and magnitude of all forces in the table below. Use value of the mass of the holder plus the additional mass as the magnitude. Do not include units in the recordings.
4. Calculate $x$ - and $y$-components of Force 1, Force 2, and Force 3; afterward, total each corresponding column to find $x$ - and $y$ - components of the Net Force (or Resultant).
5. Calculate the magnitude and direction of the Net Force (or Resultant).
6. Since the disk is balanced, the magnitude of the Net Force is expected to be zero. Comment on degree of achievement in the experiment. (Example: The net force is (small/comparable) to the size of the vectors, so it (is/isn't) close to zero as expected.)

| Force | Magnitude | Direction ($)$ | x-component | y-component |
| :---: | :---: | :---: | :---: | :---: |
| Force $\vec{F}_{1}$ |  |  |  |  |
| Force $\vec{F}_{2}$ |  |  |  |  |
| Force $\overrightarrow{F_{3}}$ |  |  |  |  |
| Net Force <br> (or Resultant) |  |  |  |  |

Table 4. Measurements and calculations for three none perpendicular forces of different magnitudes.

## Requirements for the Report (also consult the rubric):

Save your Excel files through your Blackboard Group File Exchange

- The abstract section must contain the following explanations in paragraph form:
- The general experimental set up (think about what remained the same in all 4 parts)
- Specifics in the setup of each part
- The calculations (x-components, y-components, magnitudes, directions, resultant, neutralizer)
- Define mathematically or in words the relationship between resultant and neutralizer
- How each part supported the condition of static equilibrium (define) based on \% difference (they all should have been very small).
- If there was a \% difference, what factors did we not consider that may have contributed to this difference?
- The data section must include
- 4 Tables (labeled and captioned)

